ALGEBRA

1. Without using long division, find the remainder when $x^3 - 5x^2 + 7$ is divided by $(x-1)^2$.

2. If $4x^3 + kx^2 + px + 2$ is divisible by $x^2 + a^2$, prove that $kp - 8 = 0$.

3. Given that the equation has repeated roots, solve the equation: $18x^3 + 3x^2 - 88x - 80 = 0$

4. Solve the equations:
$$2\log_y x + 2\log_x y = 5$$
$$xy = 27$$

5a) Solve the inequality: $|x-2| > 3|2x+1|$

b) Solve the inequality: $3x + 5 \le \dfrac{6}{2-x}$

6. When $(1+2x)(1+ax)^{2/3}$ where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

i) Find the value of a.

ii) When a has this value, find the term in x^3 in the expansion of $(1+2x)(1+ax)^{2/3}$, simplifying the coefficient.

7. The variables x and y satisfy the equation $x^n y = C$, where n and C are constants. When $x = 1.10$, $y = 5.20$ and when $x = 3.20$, $y = 1.05$. Find the values of n and C.

8. The polynomial $f(x)$ is defined by $f(x) \equiv 12x^3 + 25x^2 - 4x - 12$. Show that $f(-2) = 0$ and factorise $f(x)$ completely, hence, given that $12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0$ state the value of 3^y and hence find y correct to 3 significant figures.

9. Express $\left(-1-\sqrt{3}i\right)^6$ in the form $x + iy$.

10. Describe the locus represented by $2|z-2i| = 3|z+1|$ and hence state the centre and radius.

11. In an attempt to save her sinking business, Kevin borrowed shs. 100million from a money launder on the terms that Kevin was to pay a fixed amount of money at the end of every year for a period of 5 years and an interest of 10% p.a was to be charged on all money's owed. Find how much money she was to pay at the end of every year provided she does not default.

12. The sum of the first p terms of an arithmetic progression is q and the sum of the first q terms is p. Find the sum of the first $p+q$ terms.

13. The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, show that the last term is 5120.

14. Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$.

15. Given that the first three terms in the expansion in ascending powers of x of $(1-8x)^{1/4}$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1+bx}\right)$, find the values of a and b. Hence, find an approximation to $(0.6)^{1/4}$ in the form p/q.

16. Prove by induction that $8^n - 7n + 6$ is divisible by 7 for all $n \geq 1$.

ANALYSIS

17. Given that $x^n + y^n = 1$, show that $\frac{d^2y}{dx^2} = \frac{(n-1)x^{n-2}}{y^{2n-1}}$.

18. Express $f(x) \equiv \frac{x^2+3x+3}{(x+1)(x+3)}$ in partial fractions, hence, show that $\int_0^3 \frac{x^2+3x+3}{(x+1)(x+3)}dx = 3 - \frac{1}{2}In2$

19. Show that $\int_0^{\pi} x^2 \sin x \, dx = \pi^2 - 4$

20. Prove the identity: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, hence, find the exact value of $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3\theta \, d\theta$.

21. The variables x and y are related by the differential equation $x\frac{dy}{dx} = 1 - y^2$. When $x = 2,\ y = 0$. Solve the differential equation, obtaining an expression for y in terms of x.

22. A hemispherical bowl is being filled with water at a uniform rate. When the height of the water is $h\,cm$ the volume is $\pi\left(rh^2 - \frac{1}{3}h^3\right)cm^3$, $r\,cm$ being the radius of the hemisphere. Find the rate at which the water level is rising when it is half way to the top, given that $r = 6\,cm$ and the bowl fills in $1\min$.

23. Sand is pouring at a constant rate of $\frac{1000\pi}{3}cm^3s^{-1}$ forms a right circular conical heap of height $10\,cm$. Express the volume of heap in terms of the vertical angle, hence, find the rate of change of the vertical angle when it is 60^o.

24. Given that $y = f(x)$ is the equation of a curve, such that $f(1) = 2$ and $\frac{f(x+h) - f(x)}{h} = 3(x+h)^2 - 2(x+h)$; find the equation of the curve.

25. A right circular cylinder is inscribed in a sphere of given radius a. Prove that the total area of its surface (including its ends) is $2\pi a^2(\sin 2\theta + \cos^2\theta)$, where $a\cos\theta$ is the radius of an end. Hence prove that the maximum value of the total area is $\pi a^2(\sqrt{5}+1)$.

26. If $y = \frac{Inx^2}{\log_{x^2} 10}$, find the value of $\frac{dy}{dx}$ at $x = e$, correct to two s.f.

27. If $y = \sin 2x In(\tan x)$, show that $\frac{d^2y}{dx^2} + 4y = 4\cot 2x$.

28. The curve $y = ax^3 + bx^2 + cx$ passes through the point $(-1,\ 16)$ and has a stationary point at $(1,\ -4)$. Find a, b, c.

29. Sketch the curve $y = \frac{4x^2 - 10x + 7}{(x-1)(x-2)}$ determine the turning points and clearly state the asymptotes.

30. Use small changes to estimate $\tan 44^o 54'$.

31. On a certain curve for which $\frac{dy}{dx} = x + \frac{a}{x^2}$, the point $(2, 1)$ is a point of inflexion. Find the value of a and the equation of the curve.

32. Given that $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, prove that $\frac{dy}{dx} = \cot\frac{1}{2}\theta$ and $\frac{d^2y}{dx^2} = -\frac{1}{4a}\operatorname{cosec}^4 \frac{1}{2}\theta$.

TRIGONOMETRY

33. Solve the equation $3\sec^2\frac{x}{2} = \tan\frac{x}{2} + 5$ for $0^0 \le x \le 360^0$.

34. Two points A and B on a straight coastline are $1km$ apart, B being due East of A. If a ship is observed on bearings 167^o and 205^o from A and B respectively, what is the distance of the coastline?

35. Express $\sqrt{\dfrac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta}}$ in terms of $\tan\theta$.

36. Prove that $\tan\frac{1}{2}(A-B) + \tan\frac{1}{2}(A+\mathbf{B}) = \dfrac{2\sin A}{\cos A + \cos B}$

37. Solve for x: $\tan^{-1}x + \tan^{-1}(1-x) = \tan^{-1}\left(\dfrac{9}{7}\right)$

38. Prove that: $\tan\left(\dfrac{\pi}{4}+\theta\right) - \tan\left(\dfrac{\pi}{4}-\theta\right) = 2\tan 2\theta$

39. Express $4\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$ and hence, find the maximum and minimum values of the function $\dfrac{1}{6+4\sin x - 3\cos x}$ stating clearly the values of x.

40. Show that the equation $\tan(30^o+\theta) = 2\tan(60^o-\theta)$ can be written in the form $\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0$, hence, solve or otherwise, solve the equation $\tan(30^o+\theta) = 2\tan(60^o-\theta)$ for $0^o \le \theta \le 180^o$.

41. Prove that: $\cos A = \dfrac{s(b-a+c)}{bc} - 1$

42. Solve the equation: $5\sin^2 2x - 3\sin 2x\cos 2x - 14\cos^2 2x = 0$, for $0^o \le x \le 90^o$.

43. Given that $y = \dfrac{\cos x - 2\cos 2x + \cos 3x}{\cos x + 2\cos 2x + \cos 3x}$, prove that $y = -\tan^2\dfrac{x}{2}$, hence, simplify $\tan^2 15^o$ in the form $p + q\sqrt{r}$ where p, q, r are integers.

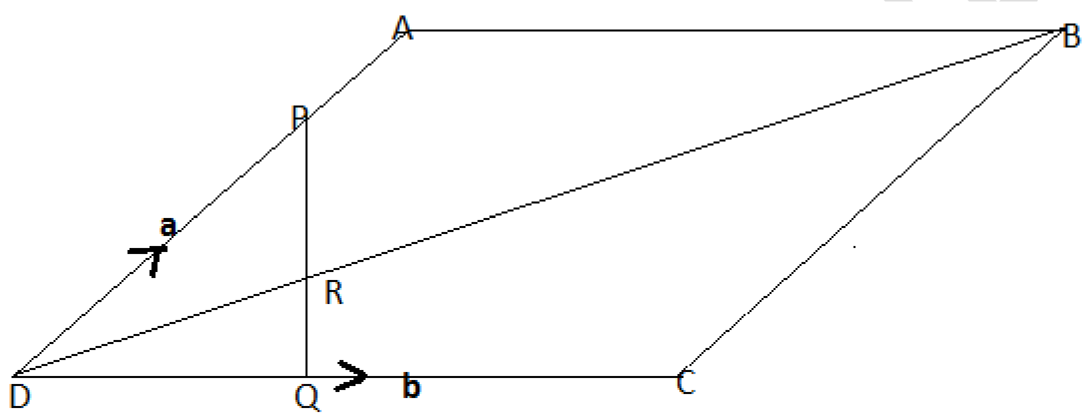
VECTORS

44. With respect to the origin O, the points P, Q, R, S have position vectors given by $\vec{OP} = \mathbf{i} - \mathbf{k}$, $\vec{OQ} = -2\mathbf{i} + 4\mathbf{j}$, $\vec{OR} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\vec{OS} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$.

i) Find the equation of the plane containing P, Q and R, giving your answer in the form $ax + by + cz = d$.

ii) The point N is the foot of the perpendicular from S to this plane. Find the position vector of N and show that the length of SN is 7.

45. The parallelogram $ABCD$ shows the points P and Q dividing each of the lines **AD** and **DC** in the ratio $1 : 4$.



a) If $\mathbf{DA} = \mathbf{a}$ and $\mathbf{DC} = \mathbf{b}$, express the following vectors in terms of $\mathbf{a}$ and $\mathbf{b}$.

i) **AC** ii) **CB** iii) **DB**

b)i) Find the ratio in which R divides **DB**.

ii) Find the ratio in which R divides **PQ**.

46. The straight line l passes through the points with coordinates $(-5,\ 3,\ 6)$ and $(5,\ 8,\ 1)$. The plane p has equation $2x - y + 4z = 9$.

i) Find the coordinates of the point of intersection of l and p.

ii) Find the acute angle between l and p.

47a) P and Q are points with position vectors $4i - 3j + 5k$ and $i + 2k$ respectively. Find the coordinates of the point R such that PQ : PR = 2 : 1.

b) If the vector $5\mathbf{i} - \lambda\mathbf{j} + \mathbf{k}$ is perpendicular to the line $r = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$, find the value of λ.

c) Determine the equation of the plane through the point $(5,-2,3)$ and the perpendicular vector $\overrightarrow{AB}$, where $A=(2,-3,3)$ and $B=(5,1,2)$.

48. Given that $|p|=5$, $|q|=10$ and $p\,.\,q=22$. Determine:

i) the angle between p and q.

ii) $|p+q|$ and $|p-q|$

49a) Show that the vectors $a=2i+3j+4k,\ b=-i+2j-3k,\ c=i+5j+k$ form a triangle and determine the area of this triangle.

b) A line L passes through the point $(1,\ 4,\ 0)$ and is parallel to the vector $r=2i-j+5k$.

Determine: i) the point of intersection of L and the line $\frac{x-1}{4}=\frac{y-4}{3}=z$

ii) the equation of a plane that contains m and the point $(1,\ 1,\ 5)$.

GEOMETRY

50. Find the possible loci of the points that are equidistant from the lines $6y=7x+1$ and $9x+2y+3=0$.

51. Show that the equation $y^2-4y=4x$ represents a parabola and make a sketch of the curve.

52. Find the equation of the parabola with focus (2, 1) and directrix $x+y=2$.

53. Show that if the line $y=mx+c$ touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $c^2=a^2m^2+b^2$. Find the equations of the tangents to the ellipse $4x^2+9y^2=1$, which are perpendicular to $y=2x+3$.

54. Find the area of a triangle ABC, with vertices $A(3,\ 2)$, $B(5,\ -6)$, and $C(-3,\ -3)$.

55. The distance of the point (2, – 1) from the line $y=\frac{3}{4}x+c$ is twice its distance from the line $y=-4/3\,x$. Find the value of c.

56. The gradient of the side PQ of the rectangle $PQRS$ is $3/4$. The coordinates of the opposite corners Q, S are respectively $(6,\ 3)$ and $(-5,\ 1)$. Find the equation of PR.

END